# условная оптимизация проверка линейных ограничений.

Лекция 13

# метод наименьших квадратов с линейными ограничениями

$$\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon};\ \mathbf{H}\boldsymbol{\beta} = \mathbf{r}$$

$$Ex1: \quad Y = C + I + G + \varepsilon; \quad C = cY \Rightarrow Y = \frac{1}{1-c} I + \frac{1}{1-c} G + \tilde{\varepsilon}$$

$$\Rightarrow Y = \alpha I + \beta G + \tilde{\varepsilon}, \quad \alpha - \beta = 0 \Rightarrow \mathbf{H} = \begin{pmatrix} 1 & \text{-}1 \end{pmatrix}; \mathbf{r} = 0$$

$$Ex2: \quad Y = H^{\alpha} K^{\beta} L^{\gamma} \varepsilon \Rightarrow \ln Y = \alpha H + \beta \ln K + \gamma \ln L + \tilde{\varepsilon}, \quad \begin{cases} \alpha + \beta + \gamma = 1 \\ \beta + \gamma = 1/2 \end{cases} \Rightarrow$$

$$\Rightarrow \mathbf{H} = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix}; \mathbf{r} = \begin{pmatrix} 1 \\ 1/2 \end{pmatrix}$$

$$\sum_{i=1}^{n} e_i^2 = \mathbf{e}'\mathbf{e} = \left(\mathbf{y} - \mathbf{X}\boldsymbol{\beta}\right)' \left(\mathbf{y} - \mathbf{X}\boldsymbol{\beta}\right) \xrightarrow[\mathbf{H}\boldsymbol{\beta} = \mathbf{r}]{} \min$$

**метод наименьших квадратов с линейными ограничениями**

$$\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon};\ \mathbf{H}\boldsymbol{\beta} = \mathbf{r}$$

$$\boldsymbol{\beta} = \left(\mathbf{X}'\mathbf{X}\right)^{-1}\mathbf{X}'\left(\mathbf{y} - \boldsymbol{\varepsilon}\right) \Rightarrow$$

$$\Rightarrow \mathbf{r} = \mathbf{H}\boldsymbol{\beta} = \mathbf{H}\left(\mathbf{X}'\mathbf{X}\right)^{-1}\mathbf{X}'\left(\mathbf{y} - \boldsymbol{\varepsilon}\right) \Rightarrow$$

$$\Rightarrow \mathbf{A}\boldsymbol{\varepsilon} = \mathbf{b}; \quad \mathbf{A} = \mathbf{H}\left(\mathbf{X}'\mathbf{X}\right)^{-1}\mathbf{X}',\ \mathbf{b} = \mathbf{H}\left(\mathbf{X}'\mathbf{X}\right)^{-1}\mathbf{X}'\mathbf{y} - \mathbf{r}$$

Обычный МНК не проходит!!!!

**метод наименьших квадратов с линейными ограничениями**

$$\mathbf{y} = \mathbf{X\beta} + \boldsymbol{\varepsilon};\ \mathbf{H\beta} = \mathbf{r}$$

$$\sum_{i=1}^{n} e_i^2 = \mathbf{e}'\mathbf{e} = \left(\mathbf{y} - \mathbf{X\beta}\right)' \left(\mathbf{y} - \mathbf{X\beta}\right) \xrightarrow[\mathbf{H\beta}=\mathbf{r}]{} \min$$

*Утв.* $\hat{\boldsymbol{\beta}}_R = \hat{\boldsymbol{\beta}}_{UR} - \left(\mathbf{X}'\mathbf{X}\right)^{-1}\mathbf{H}'\left[\mathbf{H}\left(\mathbf{X}'\mathbf{X}\right)^{-1}\mathbf{H}'\right]^{-1}\left(\mathbf{H}\hat{\boldsymbol{\beta}}_{UR} - \mathbf{r}\right)$

$$\hat{\boldsymbol{\beta}}_{UR} = \left(\mathbf{X}'\mathbf{X}\right)^{-1}\mathbf{X}'\mathbf{y};$$

# УСЛОВНАЯ ОПТИМИЗАЦИЯ

$$\mathbf{y} = \mathbf{X\beta} + \boldsymbol{\varepsilon};\ \mathbf{H\beta} = \mathbf{r}$$

$$\sum_{i=1}^{n} e_i^2 = \mathbf{e}'\mathbf{e} = \left(\mathbf{y} - \mathbf{X\beta}\right)'\left(\mathbf{y} - \mathbf{X\beta}\right) \xrightarrow[\mathbf{H\beta}=\mathbf{r}]{} \min$$

$$\mathbf{L}\left(\boldsymbol{\beta};\boldsymbol{\lambda}\right) = \left(\mathbf{y} - \mathbf{X\beta}\right)'\left(\mathbf{y} - \mathbf{X\beta}\right) - \boldsymbol{\lambda}'\left(\mathbf{H\beta} - \mathbf{r}\right) \to \min_{\boldsymbol{\beta};\boldsymbol{\lambda}}$$

# УСЛОВНАЯ ОПТИМИЗАЦИЯ

$$\mathbf{L}(\boldsymbol{\beta};\boldsymbol{\lambda}) = (\mathbf{y}-\mathbf{X}\boldsymbol{\beta})'(\mathbf{y}-\mathbf{X}\boldsymbol{\beta}) - \boldsymbol{\lambda}'(\mathbf{H}\boldsymbol{\beta}-\mathbf{r}) \to \min_{\boldsymbol{\beta};\boldsymbol{\lambda}}$$

$FOC:$

$\mathbf{L}'_{\beta_i}(\boldsymbol{\beta};\boldsymbol{\lambda}) = 0\ \forall j$ k уравнений (по кол-ву коэф-тов)

$\mathbf{L}'_{\lambda_j}(\boldsymbol{\beta};\boldsymbol{\lambda}) = 0\ \forall j$ m уравнений (по кол-ву условий)

$SOC: \dfrac{\partial^2 \mathbf{L}(\boldsymbol{\beta};\boldsymbol{\lambda})}{\partial\boldsymbol{\beta}'\partial\boldsymbol{\lambda}} \geq 0$ (в смысле положительной определенности)

# УСЛОВНАЯ ОПТИМИЗАЦИЯ

$$\mathbf{L}(\boldsymbol{\beta};\boldsymbol{\lambda}) = \mathbf{y}'\mathbf{y} - \mathbf{y}'\mathbf{X}\boldsymbol{\beta} - \boldsymbol{\beta}'\mathbf{X}'\mathbf{y} + \boldsymbol{\beta}'\mathbf{X}'\mathbf{X}\boldsymbol{\beta} - \boldsymbol{\lambda}'\mathbf{H}\boldsymbol{\beta} + \boldsymbol{\lambda}'\mathbf{r}$$

$$FOC: \quad \begin{cases} \mathbf{L}'_{\boldsymbol{\beta}'}(\boldsymbol{\beta};\boldsymbol{\lambda}) = \mathbf{0} \\ \mathbf{L}'_{\boldsymbol{\lambda}}(\boldsymbol{\beta};\boldsymbol{\lambda}) = \mathbf{0} \end{cases} \Leftrightarrow \begin{cases} \mathbf{L}'_{\boldsymbol{\beta}'}(\boldsymbol{\beta};\boldsymbol{\lambda}) = -2\mathbf{X}'\mathbf{y} + 2\mathbf{X}'\mathbf{X}\boldsymbol{\beta} - \mathbf{H}'\boldsymbol{\lambda} = \mathbf{0} & (1) \\ \mathbf{L}'_{\boldsymbol{\lambda}}(\boldsymbol{\beta};\boldsymbol{\lambda}) = \mathbf{r} - \mathbf{H}\boldsymbol{\beta} = \mathbf{0} & (2) \end{cases}$$

$$SOC: \frac{\partial^2 \mathbf{L}(\boldsymbol{\beta};\boldsymbol{\lambda})}{\partial\boldsymbol{\beta}'\partial\boldsymbol{\lambda}} = \begin{pmatrix} 2(\mathbf{X}'\mathbf{X})' & -\mathbf{H}' \\ -\mathbf{H}' & 0 \end{pmatrix} \geq 0$$

# УСЛОВНАЯ ОПТИМИЗАЦИЯ

$$\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon};\ \mathbf{H}\boldsymbol{\beta} = \mathbf{r}$$

$$FOC: \quad \begin{cases} \mathbf{L}'_{\boldsymbol{\beta}}\left(\boldsymbol{\beta};\boldsymbol{\lambda}\right) = \mathbf{0} \\ \mathbf{L}'_{\boldsymbol{\lambda}}\left(\boldsymbol{\beta};\boldsymbol{\lambda}\right) = \mathbf{0} \end{cases} \Leftrightarrow \begin{cases} \mathbf{L}'_{\boldsymbol{\beta}'}\left(\boldsymbol{\beta};\boldsymbol{\lambda}\right) = -2\mathbf{X}'\mathbf{y} + 2\mathbf{X}'\mathbf{X}\boldsymbol{\beta} - \mathbf{H}'\boldsymbol{\lambda} = \mathbf{0} & (1) \\ \mathbf{L}'_{\boldsymbol{\lambda}}\left(\boldsymbol{\beta};\boldsymbol{\lambda}\right) = \mathbf{r} - \mathbf{H}\boldsymbol{\beta} = \mathbf{0} & (2) \end{cases}$$

$$(1) \Rightarrow \boldsymbol{\beta} = \left(\mathbf{X}'\mathbf{X}\right)^{-1}\mathbf{X}'\mathbf{y} + 0.5\left(\mathbf{X}'\mathbf{X}\right)^{-1}\mathbf{H}'\boldsymbol{\lambda} \quad (3)$$

# УСЛОВНАЯ ОПТИМИЗАЦИЯ

$$\mathbf{y} = \mathbf{X\beta} + \boldsymbol{\varepsilon};\ \mathbf{H\beta} = \mathbf{r}$$

$$FOC: \quad \begin{cases} \mathbf{L}'_{\boldsymbol\beta}(\boldsymbol\beta;\boldsymbol\lambda) = \mathbf{0} \\ \mathbf{L}'_{\boldsymbol\lambda}(\boldsymbol\beta;\boldsymbol\lambda) = \mathbf{0} \end{cases} \Leftrightarrow \begin{cases} \mathbf{L}'_{\boldsymbol\beta'}(\boldsymbol\beta;\boldsymbol\lambda) = -2\mathbf{X}'\mathbf{y} + 2\mathbf{X}'\mathbf{X}\boldsymbol\beta - \mathbf{H}'\boldsymbol\lambda = \mathbf{0} \quad (1) \\ \mathbf{L}'_{\boldsymbol\lambda}(\boldsymbol\beta;\boldsymbol\lambda) = \mathbf{r} - \mathbf{H}\boldsymbol\beta = \mathbf{0} \quad (2) \end{cases}$$

$$(1) \Rightarrow \boldsymbol\beta = \left(\mathbf{X}'\mathbf{X}\right)^{-1}\mathbf{X}'\mathbf{y} + 0.5\left(\mathbf{X}'\mathbf{X}\right)^{-1}\mathbf{H}'\boldsymbol\lambda \quad (3)$$

$$(2),(3) \Rightarrow \mathbf{r} = \mathbf{H}\boldsymbol\beta = \mathbf{H}\hat{\boldsymbol\beta}_{UR} + 0.5\mathbf{H}\left(\mathbf{X}'\mathbf{X}\right)^{-1}\mathbf{H}'\boldsymbol\lambda \quad (4)$$

# УСЛОВНАЯ ОПТИМИЗАЦИЯ

$$\mathbf{y} = \mathbf{X\beta} + \boldsymbol{\varepsilon};\ \mathbf{H\beta} = \mathbf{r}$$

$$FOC:\quad \begin{cases} \mathbf{L}'_{\boldsymbol\beta}\left(\boldsymbol\beta;\boldsymbol\lambda\right) = \mathbf{0} \\ \mathbf{L}'_{\boldsymbol\lambda}\left(\boldsymbol\beta;\boldsymbol\lambda\right) = \mathbf{0} \end{cases} \Leftrightarrow \begin{cases} \mathbf{L}'_{\boldsymbol\beta'}\left(\boldsymbol\beta;\boldsymbol\lambda\right) = -2\mathbf{X}'\mathbf{y} + 2\mathbf{X}'\mathbf{X}\boldsymbol\beta - \mathbf{H}'\boldsymbol\lambda = \mathbf{0} & (1) \\ \mathbf{L}'_{\boldsymbol\lambda}\left(\boldsymbol\beta;\boldsymbol\lambda\right) = \mathbf{r} - \mathbf{H}\boldsymbol\beta = \mathbf{0} & (2) \end{cases}$$

$$(1) \Rightarrow \boldsymbol\beta = \left(\mathbf{X}'\mathbf{X}\right)^{-1}\mathbf{X}'\mathbf{y} + 0.5\left(\mathbf{X}'\mathbf{X}\right)^{-1}\mathbf{H}'\boldsymbol\lambda \quad (3)$$

$$(2),(3) \Rightarrow \mathbf{r} = \mathbf{H}\boldsymbol\beta = \mathbf{H}\hat{\boldsymbol\beta}_{UR} + 0.5\mathbf{H}\left(\mathbf{X}'\mathbf{X}\right)^{-1}\mathbf{H}'\boldsymbol\lambda \quad (4)$$

$$(4) \Rightarrow \hat{\boldsymbol\lambda} = 2\left[\mathbf{H}\left(\mathbf{X}'\mathbf{X}\right)^{-1}\mathbf{H}'\right]^{-1}\left(\mathbf{r} - \mathbf{H}\hat{\boldsymbol\beta}_{UR}\right) \quad (5)$$

# УСЛОВНАЯ ОПТИМИЗАЦИЯ

$$\mathbf{y} = \mathbf{X\beta} + \boldsymbol{\varepsilon};\ \mathbf{H\beta} = \mathbf{r}$$

$$FOC: \quad \begin{cases} \mathbf{L}'_{\boldsymbol{\beta}}(\boldsymbol{\beta};\boldsymbol{\lambda}) = \mathbf{0} \\ \mathbf{L}'_{\boldsymbol{\lambda}}(\boldsymbol{\beta};\boldsymbol{\lambda}) = \mathbf{0} \end{cases} \Leftrightarrow \begin{cases} \mathbf{L}'_{\boldsymbol{\beta}'}(\boldsymbol{\beta};\boldsymbol{\lambda}) = -2\mathbf{X}'\mathbf{y} + 2\mathbf{X}'\mathbf{X\beta} - \mathbf{H}'\boldsymbol{\lambda} = \mathbf{0} & (1) \\ \mathbf{L}'_{\boldsymbol{\lambda}}(\boldsymbol{\beta};\boldsymbol{\lambda}) = \mathbf{r} - \mathbf{H\beta} = \mathbf{0} & (2) \end{cases}$$

$$(1) \Rightarrow \boldsymbol{\beta} = \left(\mathbf{X}'\mathbf{X}\right)^{-1}\mathbf{X}'\mathbf{y} + 0.5\left(\mathbf{X}'\mathbf{X}\right)^{-1}\mathbf{H}'\boldsymbol{\lambda} \quad (3)$$

$$(4) \Rightarrow \hat{\boldsymbol{\lambda}} = 2\left[\mathbf{H}\left(\mathbf{X}'\mathbf{X}\right)^{-1}\mathbf{H}'\right]^{-1}\left(\mathbf{r} - \mathbf{H}\hat{\boldsymbol{\beta}}_{UR}\right) \quad (5)$$

$$(1),(5) \Rightarrow \hat{\boldsymbol{\beta}}_R = \hat{\boldsymbol{\beta}}_{UR} + \left(\mathbf{X}'\mathbf{X}\right)^{-1}\mathbf{H}'\left[\mathbf{H}\left(\mathbf{X}'\mathbf{X}\right)^{-1}\mathbf{H}'\right]^{-1}\left(\mathbf{r} - \mathbf{H}\hat{\boldsymbol{\beta}}_{UR}\right)$$

**метод наименьших квадратов с линейными ограничениями**

$$\mathbf{y} = \mathbf{X\beta} + \boldsymbol{\varepsilon};\ \mathbf{H\beta} = \mathbf{r}$$

$$\sum_{i=1}^{n} e_i^2 = \mathbf{e}'\mathbf{e} = \left(\mathbf{y} - \mathbf{X\beta}\right)'\left(\mathbf{y} - \mathbf{X\beta}\right) \xrightarrow[\mathbf{H\beta}=\mathbf{r}]{} \min$$

*Утв.* $\hat{\boldsymbol{\beta}}_R = \hat{\boldsymbol{\beta}}_{UR} - \left(\mathbf{X}'\mathbf{X}\right)^{-1}\mathbf{H}'\left[\mathbf{H}\left(\mathbf{X}'\mathbf{X}\right)^{-1}\mathbf{H}'\right]^{-1}\left(\mathbf{H}\hat{\boldsymbol{\beta}}_{UR} - \mathbf{r}\right)$

$$\hat{\boldsymbol{\beta}}_{UR} = \left(\mathbf{X}'\mathbf{X}\right)^{-1}\mathbf{X}'\mathbf{y};$$

# ПРОВЕРКА ГИПОТЕЗЫ О ЛИНЕЙНЫХ ОГРАНИЧЕНИЯХ

$$\hat{\boldsymbol{\beta}}_R = \hat{\boldsymbol{\beta}}_{UR} + \left(\mathbf{X}'\mathbf{X}\right)^{-1}\mathbf{H}'\left[\mathbf{H}\left(\mathbf{X}'\mathbf{X}\right)^{-1}\mathbf{H}'\right]^{-1}\left(\mathbf{r} - \mathbf{H}\hat{\boldsymbol{\beta}}_{UR}\right) \Rightarrow$$

$$\Rightarrow \hat{\boldsymbol{\beta}}_R = \left(\mathbf{I} - \left(\mathbf{X}'\mathbf{X}\right)^{-1}\mathbf{H}'\left[\mathbf{H}\left(\mathbf{X}'\mathbf{X}\right)^{-1}\mathbf{H}'\right]^{-1}\mathbf{H}\right)\hat{\boldsymbol{\beta}}_{UR} + \left(\mathbf{X}'\mathbf{X}\right)^{-1}\mathbf{H}'\left[\mathbf{H}\left(\mathbf{X}'\mathbf{X}\right)^{-1}\mathbf{H}'\right]^{-1}\mathbf{r} =$$

$$\Rightarrow \hat{\boldsymbol{\beta}}_R = \mathbf{R}_1\hat{\boldsymbol{\beta}}_{UR} + \mathbf{R}_2$$

# ПРОВЕРКА ГИПОТЕЗЫ О ЛИНЕЙНЫХ ОГРАНИЧЕНИЯХ

$$\hat{\boldsymbol{\beta}}_R = \left(\mathbf{I} - (\mathbf{X}'\mathbf{X})^{-1}\mathbf{H}'\left[\mathbf{H}(\mathbf{X}'\mathbf{X})^{-1}\mathbf{H}'\right]^{-1}\mathbf{H}\right)\hat{\boldsymbol{\beta}}_{UR} + (\mathbf{X}'\mathbf{X})^{-1}\mathbf{H}'\left[\mathbf{H}(\mathbf{X}'\mathbf{X})^{-1}\mathbf{H}'\right]^{-1}\mathbf{r}$$

$$\hat{\boldsymbol{\beta}}_R = \mathbf{R_1}\hat{\boldsymbol{\beta}}_{UR} + \mathbf{R}_2$$

$$\xi \in N\left(a;\sigma^2\right) \Rightarrow r_1\xi + r_2 \in N\left(r_1a + r_2;\sigma^2 r_1^2\right);$$

$$\boldsymbol{\xi} \in \mathbf{N}\left(\mathbf{a};\boldsymbol{\Sigma}\right) \Rightarrow \mathbf{R_1}\boldsymbol{\xi} + \mathbf{R}_2 \in \mathbf{N}\left(\mathbf{R_1 a} + \mathbf{R}_2;\mathbf{R}_1\boldsymbol{\Sigma}\mathbf{R}_1'\right)$$

$$\mathrm{Var}\,\hat{\boldsymbol{\beta}}_R = \mathbf{R_1}\,\mathrm{Var}\,\hat{\boldsymbol{\beta}}_{UR}\mathbf{R}_1' = \sigma^2\mathbf{R}_1(\mathbf{X}'\mathbf{X})^{-1}\mathbf{R}_1'$$

$$\hat{\boldsymbol{\beta}}_R \in \mathbf{N}\left(\boldsymbol{\beta};\sigma^2\mathbf{R}_1(\mathbf{X}'\mathbf{X})^{-1}\mathbf{R}_1'\right)$$

*Упражнение*: доказать: $\mathrm{E}\,\hat{\boldsymbol{\beta}}_R = \boldsymbol{\beta}$

## ПРОВЕРКА ГИПОТЕЗЫ О ЛИНЕЙНЫХ ОГРАНИЧЕНИЯХ

$$\mathbf{y} = \mathbf{X\beta} + \boldsymbol{\varepsilon}; \ \mathbf{H\beta} = \mathbf{r}$$

$$\mathbf{H}\hat{\boldsymbol{\beta}}_R = \mathbf{r}; \quad \text{if } \mathbf{H}\hat{\boldsymbol{\beta}}_{UR} = \mathbf{r} \Rightarrow \hat{\boldsymbol{\beta}}_R = \hat{\boldsymbol{\beta}}_{UR} \Rightarrow$$

$\Rightarrow$ Рассмотрим линейную регрессию $\mathbf{Y} = \mathbf{X\beta} + \boldsymbol{\varepsilon}$;

$$\hat{\boldsymbol{\beta}}_{UR} = \hat{\boldsymbol{\beta}}_{OLS}$$

и проверим гипотезу: $\mathbf{H\beta} = \mathbf{r}$

# ПРОВЕРКА ГИПОТЕЗЫ О ЛИНЕЙНЫХ ОГРАНИЧЕНИЯХ

$$\xi \in N\left(a;\sigma^2\right) \Rightarrow h\xi \in N\left(ha;\sigma^2 h^2\right);$$

$$\boldsymbol{\xi} \in \mathbf{N}\left(\mathbf{a};\boldsymbol{\Sigma}\right) \Rightarrow \mathbf{H}\boldsymbol{\xi} \in \mathbf{N}\left(\mathbf{Ha};\mathbf{H}\boldsymbol{\Sigma}\mathbf{H}'\right)$$

$$\hat{\boldsymbol{\beta}}_{OLS} \in \mathbf{N}\left(\boldsymbol{\beta};\sigma^{\mathbf{2}}\left(\mathbf{X}'\mathbf{X}\right)^{-1}\right) \Rightarrow \mathbf{H}\hat{\boldsymbol{\beta}}_{UR} \in \mathbf{N}\left(\mathbf{r};\sigma^{\mathbf{2}}\mathbf{H}\left(\mathbf{X}'\mathbf{X}\right)^{-1}\mathbf{H}'\right)$$

# ПРОВЕРКА ГИПОТЕЗЫ О ЛИНЕЙНЫХ ОГРАНИЧЕНИЯХ

$$\hat{\boldsymbol{\beta}}_{OLS} \in \mathbf{N}\left(\boldsymbol{\beta};\sigma^{\mathbf{2}}\left(\mathbf{X}'\mathbf{X}\right)^{-1}\right) \Rightarrow \mathbf{H}\hat{\boldsymbol{\beta}}_{UR} \in \mathbf{N}\left(\mathbf{r};\sigma^{\mathbf{2}}\mathbf{H}\left(\mathbf{X}'\mathbf{X}\right)^{-1}\mathbf{H}'\right)$$

$$\xi \in N\left(a;\sigma^2\right) \Rightarrow z = \left(\xi - a\right)\sigma^{-1} \in N\left(0;1\right); \quad z^2 = \left(\xi - a\right)\sigma^{-2}\left(\xi - a\right) \sim \chi_1^2$$

$$\boldsymbol{\xi} \in \mathbf{N}\left(\mathbf{a};\boldsymbol{\Sigma}\right) \Rightarrow \mathbf{z} = \boldsymbol{\Sigma}^{-1/2}\left(\boldsymbol{\xi} - \mathbf{a}\right) \in \mathbf{N}\left(\mathbf{0};\mathbf{I}\right); \quad K_q = \mathbf{z}'\mathbf{z} = \left(\boldsymbol{\xi} - \mathbf{a}\right)'\boldsymbol{\Sigma}^{-1}\left(\boldsymbol{\xi} - \mathbf{a}\right) \sim \chi_q^2$$

$$\sigma^{-1}\left[\mathbf{H}\left(\mathbf{X}'\mathbf{X}\right)^{-1}\mathbf{H}'\right]^{-\frac{1}{2}}\left(\mathbf{H}\hat{\boldsymbol{\beta}}_{UR} - \mathbf{r}\right) \in \mathbf{N}\left(\mathbf{0};\mathbf{I}\right)$$

$$K_m = \sigma^{-\mathbf{2}}\left(\mathbf{H}\hat{\boldsymbol{\beta}}_{UR} - \mathbf{r}\right)'\left[\mathbf{H}\left(\mathbf{X}'\mathbf{X}\right)^{-1}\mathbf{H}'\right]^{-1}\left(\mathbf{H}\hat{\boldsymbol{\beta}}_{UR} - \mathbf{r}\right) \sim \chi_m^2$$

$$K = \frac{s^2\left(n-k\right)}{\sigma^2} = \frac{\mathbf{e}'\mathbf{e}}{\sigma^2} \sim \chi_{n-k}^2$$

# ПРОВЕРКА ГИПОТЕЗЫ О ЛИНЕЙНЫХ ОГРАНИЧЕНИЯХ

$$\hat{\boldsymbol{\beta}}_{OLS} \in \mathbf{N}\left(\boldsymbol{\beta}; \sigma^{\mathbf{2}}\left(\mathbf{X}'\mathbf{X}\right)^{-1}\right) \Rightarrow \mathbf{H}\hat{\boldsymbol{\beta}}_{UR} \in \mathbf{N}\left(\mathbf{r}; \sigma^{\mathbf{2}}\mathbf{H}\left(\mathbf{X}'\mathbf{X}\right)^{-1}\mathbf{H}'\right)$$

$$K_m = \sigma^{-\mathbf{2}}\left(\mathbf{H}\hat{\boldsymbol{\beta}}_{UR} - \mathbf{r}\right)'\left[\mathbf{H}\left(\mathbf{X}'\mathbf{X}\right)^{-1}\mathbf{H}'\right]^{-1}\left(\mathbf{H}\hat{\boldsymbol{\beta}}_{UR} - \mathbf{r}\right) \sim \chi_m^2$$

$$K = \frac{s^2\left(n-k\right)}{\sigma^2} = \frac{\mathbf{e}'\mathbf{e}}{\sigma^2} \sim \chi_{n-k}^2$$

$$F = \frac{K_m / m}{K/\left(n-k\right)} = \frac{\left(\mathbf{H}\hat{\boldsymbol{\beta}}_{UR} - \mathbf{r}\right)'\left[\mathbf{H}\left(\mathbf{X}'\mathbf{X}\right)^{-1}\mathbf{H}'\right]^{-1}\left(\mathbf{H}\hat{\boldsymbol{\beta}}_{UR} - \mathbf{r}\right)}{\mathbf{e}'\mathbf{e}}\frac{n-k}{m} \sim F\left(m; n-k\right)$$

# ПРОВЕРКА ГИПОТЕЗЫ О ЛИНЕЙНЫХ ОГРАНИЧЕНИЯХ

$$F = \frac{\left(\mathbf{H}\hat{\boldsymbol{\beta}}_{UR} - \mathbf{r}\right)' \left[\mathbf{H}\left(\mathbf{X}'\mathbf{X}\right)^{-1}\mathbf{H}'\right]^{-1} \left(\mathbf{H}\hat{\boldsymbol{\beta}}_{UR} - \mathbf{r}\right)}{\mathbf{e}'\mathbf{e}} \frac{n-k}{m} \sim F\left(m; n-k\right)$$

$Ex: \quad \mathrm{H}_0 : \beta_1 = \ldots \beta_k = 0 \Leftrightarrow \boldsymbol{\beta} = \mathbf{0} \Rightarrow$

$$\mathbf{H} = \mathbf{I};\ \mathbf{r} = \mathbf{0}; \quad F = \frac{\hat{\boldsymbol{\beta}}'_{OLS}\left(\mathbf{X}'\mathbf{X}\right)\hat{\boldsymbol{\beta}}_{OLS} / k}{s^2 / (n-k)} \sim F\left(k; n-k\right)$$

Тест идентичен тесту на основе ЦПТ:

$$\hat{\boldsymbol{\beta}} \sim \mathrm{N}\left(\mathbf{0}; \frac{\sigma^2}{n}\mathbf{Q}^{-1}\right) \Rightarrow (\mathbf{ЦПТ}) : \mathbf{z} = \left(\mathbf{Q}^{-1}\right)^{-1/2} \frac{\hat{\boldsymbol{\beta}}}{\sigma}\sqrt{n} = \left(\mathbf{X}'\mathbf{X}\right)^{1/2} \frac{\hat{\boldsymbol{\beta}}}{\sigma} \sim \mathrm{N}\left(\mathbf{0}; \mathbf{I}\right)$$

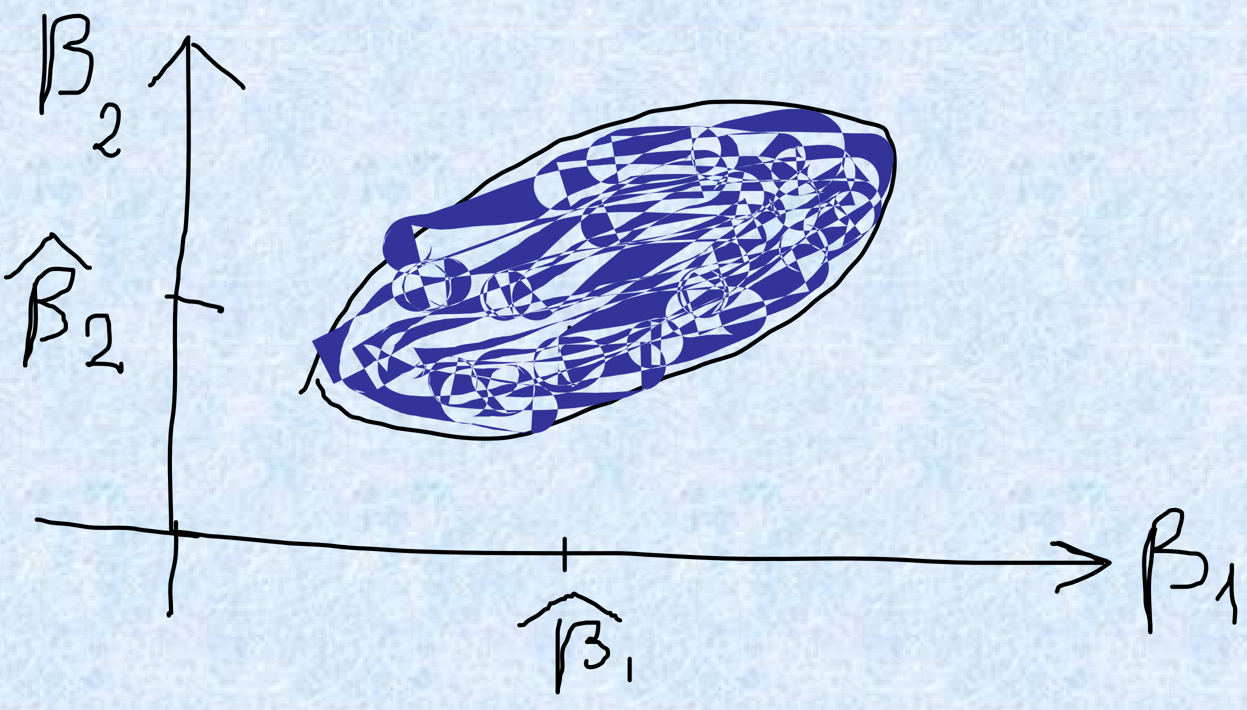
Доверительная область- эллипс в к-мерном пространстве.

*Упражнение* :

получить данный результат на основе того,

что при истинности гипотезы $\hat{\boldsymbol{\beta}}_{OLS} \in \mathbf{N}\left(\mathbf{0}; \sigma^{\mathbf{2}}\left(\mathbf{X}'\mathbf{X}\right)^{-1}\right)$

## ПОСТРОЕНИЕ ДОВЕРИТЕЛЬНЫХ ОБЛАСТЕЙ



## метод наименьших квадратов с линейными ограничениями

$$\mathbf{y} = \mathbf{X\beta} + \boldsymbol{\varepsilon};\ \mathbf{H\beta} = \mathbf{r}$$

$$\hat{\boldsymbol{\beta}}_R = \hat{\boldsymbol{\beta}}_{UR} - \left(\mathbf{X}'\mathbf{X}\right)^{-1}\mathbf{H}'\left[\mathbf{H}\left(\mathbf{X}'\mathbf{X}\right)^{-1}\mathbf{H}'\right]^{-1}\left(\mathbf{H}\hat{\boldsymbol{\beta}}_{UR} - \mathbf{r}\right)$$

$$\mathbf{e}_R = \mathbf{y} - \mathbf{X}\hat{\boldsymbol{\beta}}_R = \mathbf{Y} - \mathbf{X}\hat{\boldsymbol{\beta}}_{UR} + \mathbf{X}\left(\mathbf{X}'\mathbf{X}\right)^{-1}\mathbf{H}'\left[\mathbf{H}\left(\mathbf{X}'\mathbf{X}\right)^{-1}\mathbf{H}'\right]^{-1}\left(\mathbf{H}\hat{\boldsymbol{\beta}}_{UR} - \mathbf{r}\right) \Rightarrow$$

$$\Rightarrow \mathbf{e}_R = \mathbf{e}_{UR} + \mathbf{m}_{\mathbf{X}} \Rightarrow$$

$$\Rightarrow \mathbf{e}'_R\mathbf{e}_R = \mathbf{e}'_{UR}\mathbf{e}_{UR} + 2\mathbf{e}'_{UR}\mathbf{m}_{\mathbf{X}} + \mathbf{m}'_{\mathbf{X}}\mathbf{m}_{\mathbf{X}}$$

$$\mathbf{e}_{UR} \perp \mathbf{X} \Rightarrow \mathbf{e}'_{UR}\mathbf{m}_{\mathbf{X}} = 0 \Rightarrow$$

$$\Rightarrow \mathbf{e}'_R\mathbf{e}_R = \mathbf{e}'_{UR}\mathbf{e}_{UR} + \mathbf{m}'_{\mathbf{X}}\mathbf{m}_{\mathbf{X}};$$

## метод наименьших квадратов с линейными ограничениями

$$\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon};\ \mathbf{H}\boldsymbol{\beta} = \mathbf{r}$$

$$\hat{\boldsymbol{\beta}}_R = \hat{\boldsymbol{\beta}}_{UR} - \left(\mathbf{X}'\mathbf{X}\right)^{-1}\mathbf{H}'\left[\mathbf{H}\left(\mathbf{X}'\mathbf{X}\right)^{-1}\mathbf{H}'\right]^{-1}\left(\mathbf{H}\hat{\boldsymbol{\beta}}_{UR} - \mathbf{r}\right)$$

$$\mathbf{e}_R = \mathbf{e}_{UR} + \mathbf{m}_{\mathbf{X}}$$

$$\mathbf{e}_R = \mathbf{y} - \mathbf{X}\hat{\boldsymbol{\beta}}_R$$

$$\mathbf{e}_{UR} = \mathbf{Y} - \mathbf{X}\hat{\boldsymbol{\beta}}_{UR}$$

$$\mathbf{m}_{\mathbf{X}} = \mathbf{X}\left(\mathbf{X}'\mathbf{X}\right)^{-1}\mathbf{H}'\left[\mathbf{H}\left(\mathbf{X}'\mathbf{X}\right)^{-1}\mathbf{H}'\right]^{-1}\left(\mathbf{H}\hat{\boldsymbol{\beta}}_{UR} - \mathbf{r}\right)$$

$$\mathbf{e}'_R\mathbf{e}_R = \mathbf{e}'_{UR}\mathbf{e}_{UR} + \mathbf{m}'_{\mathbf{X}}\mathbf{m}_{\mathbf{X}}$$

$$\mathbf{m}'_{\mathbf{X}}\mathbf{m}_{\mathbf{X}} = \left(\mathbf{H}\hat{\boldsymbol{\beta}}_{UR} - \mathbf{r}\right)'\left[\mathbf{H}\left(\mathbf{X}'\mathbf{X}\right)^{-1}\mathbf{H}'\right]^{-1}\mathbf{H}\left(\mathbf{X}'\mathbf{X}\right)^{-1}\mathbf{X}'\mathbf{X}\left(\mathbf{X}'\mathbf{X}\right)^{-1}\mathbf{H}'\left[\mathbf{H}\left(\mathbf{X}'\mathbf{X}\right)^{-1}\mathbf{H}'\right]^{-1}\left(\mathbf{H}\hat{\boldsymbol{\beta}}_{UR} - \mathbf{r}\right) =$$

$$= \left(\mathbf{H}\hat{\boldsymbol{\beta}}_{UR} - \mathbf{r}\right)'\left[\mathbf{H}\left(\mathbf{X}'\mathbf{X}\right)^{-1}\mathbf{H}'\right]^{-1}\left(\mathbf{H}\hat{\boldsymbol{\beta}}_{UR} - \mathbf{r}\right) = \mathbf{e}'_R\mathbf{e}_R - \mathbf{e}'_{UR}\mathbf{e}_{UR}$$

## метод наименьших квадратов с линейными ограничениями

$$\mathbf{y} = \mathbf{X\beta} + \boldsymbol{\varepsilon};\ \mathbf{H\beta} = \mathbf{r}$$

$$\mathbf{e}'_R\mathbf{e}_R = \mathbf{e}'_{UR}\mathbf{e}_{UR} + \mathbf{m}'_{\mathbf{X}}\mathbf{m}_{\mathbf{X}}$$

$$\mathbf{m}'_{\mathbf{X}}\mathbf{m}_{\mathbf{X}} = \left(\mathbf{H}\hat{\boldsymbol{\beta}}_{UR} - \mathbf{r}\right)'\left[\mathbf{H}\left(\mathbf{X}'\mathbf{X}\right)^{-1}\mathbf{H}'\right]^{-1}\left(\mathbf{H}\hat{\boldsymbol{\beta}}_{UR} - \mathbf{r}\right) = \mathbf{e}'_R\mathbf{e}_R - \mathbf{e}'_{UR}\mathbf{e}_{UR}$$

$$F = \frac{\left(\mathbf{H}\hat{\boldsymbol{\beta}}_{UR} - \mathbf{r}\right)'\left[\mathbf{H}\left(\mathbf{X}'\mathbf{X}\right)^{-1}\mathbf{H}'\right]^{-1}\left(\mathbf{H}\hat{\boldsymbol{\beta}}_{UR} - \mathbf{r}\right)}{\mathbf{e}'\mathbf{e}}\frac{n-k}{m} =$$

$$= \frac{\left(\mathbf{e}'_R\mathbf{e}_R - \mathbf{e}'_{UR}\mathbf{e}_{UR}\right)/m}{\mathbf{e}'_{UR}\mathbf{e}_{UR}/(n-k)} = \frac{\left(\frac{ResSS_R}{TSS} - \frac{ResSS_{UR}}{TSS}\right)\Big/m}{\frac{ResSS_{UR}}{TSS}\Big/(n-k)} =$$

**метод наименьших квадратов с линейными ограничениями**

$$\mathbf{y} = \mathbf{X\beta} + \boldsymbol{\varepsilon};\ \mathbf{H\beta} = \mathbf{r}$$

$$F = \frac{\left(\mathbf{H}\hat{\boldsymbol{\beta}}_{UR} - \mathbf{r}\right)'\left[\mathbf{H}\left(\mathbf{X}'\mathbf{X}\right)^{-1}\mathbf{H}'\right]^{-1}\left(\mathbf{H}\hat{\boldsymbol{\beta}}_{UR} - \mathbf{r}\right)}{\mathbf{e}'\mathbf{e}}\frac{n-k}{m} =$$

$$= \frac{\left(\mathbf{e}'_R\mathbf{e}_R - \mathbf{e}'_{UR}\mathbf{e}_{UR}\right)/m}{\mathbf{e}'_{UR}\mathbf{e}_{UR}/(n-k)} = \frac{\left(\frac{ResSS_R}{TSS} - \frac{ResSS_{UR}}{TSS}\right)/m}{\frac{ResSS_{UR}}{TSS}/(n-k)} =$$

$$= \frac{\left(R^2_{UR} - R^2_R\right)/m}{\left(1 - R^2_{UR}\right)/(n-k)} \sim F(m; n-k)$$

# Оценка значимости группы переменных

Линейным ограничением называется условие линейной зависимости коэффициентов регрессии

Значимость группы переменных оценивается *F*-тестом

$$F = \frac{\textit{Улучшение качества уравнения} / \textit{Число использованных степеней свободы}}{\textit{Необъясненная сумма квадратов отклонений} / \textit{Оставшееся число степеней свободы}}$$

$$F = \frac{(RSS_R - RSS_{UR}) / m}{RSS_{UR} / [n-k]}$$

$$F_{кр} = F_{\alpha; m+s; n-m-s-1}$$

Значимость группы переменных не означает значимости каждой из переменных в этой группе

# *F* тест на линейные ограничения

```
. reg S ASVABC

  Source |       SS       df       MS                  Number of obs =     570
---------+------------------------------               F(  1,   568) =  284.89
   Model |  1153.80864     1  1153.80864               Prob > F      =  0.0000
Residual |  2300.43873   568  4.05006818               R-squared     =  0.3340
---------+------------------------------               Adj R-squared =  0.3329
   Total |  3454.24737   569  6.07073351               Root MSE      =  2.0125

------------------------------------------------------------------------------
       S |      Coef.   Std. Err.       t     P>|t|       [95% Conf. Interval]
---------+--------------------------------------------------------------------
  ASVABC |   .1545378   .0091559     16.879   0.000       .1365543    .1725213
   _cons |   5.770845   .4668473     12.361   0.000       4.853888    6.687803
------------------------------------------------------------------------------
```

# *F* тест на линейные ограничения

```
. reg S ASVABC SM SF

  Source |       SS       df       MS              Number of obs =     570
---------+------------------------------           F(  3,   566) =  110.83
   Model |  1278.24153     3  426.080508           Prob > F      =  0.0000
Residual |  2176.00584   566  3.84453329           R-squared     =  0.3700
---------+------------------------------           Adj R-squared =  0.3667
   Total |  3454.24737   569  6.07073351           Root MSE      =  1.9607

------------------------------------------------------------------------------
       S |      Coef.   Std. Err.       t     P>|t|       [95% Conf. Interval]
---------+--------------------------------------------------------------------
  ASVABC |   .1295006   .0099544     13.009   0.000       .1099486    .1490527
      SM |    .069403   .0422974      1.641   0.101      -.013676     .152482
      SF |   .1102684   .0311948      3.535   0.000       .0489967    .1715401
   _cons |   4.914654   .5063527      9.706   0.000       3.920094    5.909214
------------------------------------------------------------------------------
```

**Проверим на основе *F* теста совместную незначимость количества лет обучения родителей**

# *F* тест на линейные ограничения

$$Y = \beta_1 + \beta_2 X_2 + u \qquad RSS_1$$

$$Y = \beta_1 + \beta_2 X_2 + \beta_3 X_3 + \beta_4 X_4 + u \qquad RSS_2$$

$$H_0 : \beta_3 = \beta_4 = 0$$

$$H_1 : \beta_3 \neq 0 \text{ or } \beta_4 \neq 0 \text{ or both } \beta_3 \text{ and } \beta_4 \neq 0$$

$$F(\text{cost, d.f. remaining}) = \frac{\text{improvement} \;/\; \text{cost}}{\text{remaining unexplained} \;\Big/\; \text{degrees of freedom remaining}}$$

$$F(2, 570-4) = \frac{(RSS_1 - RSS_2)/2}{RSS_2/(570-4)} = \frac{(2300.4 - 2176.0)/2}{2176.0/566} = 16.18$$

$$F_{\text{crit},0.1\%}(2,120) = 7.32$$

# Оценка значимости группы переменных

Значимость включаемой группы переменных оценивается *F*-тестом

$$F = \frac{\text{Улучшение качества уравнения} / \text{Число использованных степеней свободы}}{\text{Необъясненная сумма квадратов отклонений} / \text{Оставшееся число степеней свободы}}$$

$$F = \frac{(RSS_m - RSS_{m+s}) / s}{RSS_{m+s} / \left[n - (m+s)\right]}$$

$$F_{кр} = F_{\alpha;m+s;n-m-s-1}$$

Значимость группы переменных не означает значимости каждой из переменных в этой группе

## метод наименьших квадратов с линейными ограничениями

$$\mathbf{y} = \mathbf{X\beta} + \boldsymbol{\varepsilon};\ \mathbf{H\beta} = \mathbf{r};\ \mathbf{X} = \left(1, \mathbf{X}_2, \ldots, \mathbf{X}_k\right)$$

$$F = \frac{\left(R_{UR}^2 - R_R^2\right)/m}{\left(1 - R_{UR}^2\right)/\left(n-k\right)} \sim F\left(m; n-k\right)$$

$Ex:$

$\mathrm{H}_0 : \beta_2 = \ldots = \beta_k = 0$

$\mathrm{H}_1 : \beta_2^2 + \ldots + \beta_k^2 \neq 0$

$$\mathrm{H}_0 \Rightarrow m = k-1, R_R^2 = 0 \Rightarrow F = \frac{R_{UR}^2/\left(k-1\right)}{\left(1 - R_{UR}^2\right)/\left(n-k\right)} \sim F\left(k-1; n-k\right)$$

# $F$ тест на линейные ограничения

$$Y = \beta_1 + \beta_2 X_2 + \ldots + \beta_k X_k + u$$

$$H_0 : \beta_2 = \ldots = \beta_k = 0$$

$$H_1 : \text{ at least one } \beta \neq 0$$

$$F(k-1, n-k) = \frac{R^2 \big/ (k-1)}{(1-R^2) \big/ (n-k)} =$$

$$= \frac{\dfrac{ESS}{TSS} \Big/ (k-1)}{\dfrac{RSS}{TSS} \Big/ (n-k)} = \frac{ESS \big/ (k-1)}{RSS \big/ (n-k)}$$

***ESS* / *TSS* is equal to $R^2$ and *RSS* / *TSS* is equal to (1 - $R^2$). (For proofs, see the last sequence in Chapter 3.)**

# *F* тест на линейные ограничения

$$S = \beta_1 + \beta_2 ASVABC + \beta_3 SM + \beta_4 SF + u$$

$$H_0: \beta_2 = \beta_3 = \beta_4 = 0$$

```
. reg S ASVABC SM SF

  Source |       SS       df       MS              Number of obs =     570
---------+------------------------------           F(  3,   566) =  110.83
   Model |  1278.24153     3  426.080508           Prob > F      =  0.0000
Residual |  2176.00584   566  3.84453329           R-squared     =  0.3700
---------+------------------------------           Adj R-squared =  0.3667
   Total |  3454.24737   569  6.07073351           Root MSE      =  1.9607

------------------------------------------------------------------------------
       S |      Coef.   Std. Err.       t     P>|t|       [95% Conf. Interval]
---------+--------------------------------------------------------------------
  ASVABC |   .1295006   .0099544     13.009   0.000       .1099486    .1490527
      SM |    .069403   .0422974      1.641   0.101       -.013676     .152482
      SF |   .1102684   .0311948      3.535   0.000       .0489967    .1715401
   _cons |   4.914654   .5063527      9.706   0.000       3.920094    5.909214
------------------------------------------------------------------------------
```

$$F_{crit,0.1\%}(3,120) = 5.78 \qquad F(3,566) = \frac{1278/3}{2176/566} = 110.8$$

## ПРОВЕРКА ГИПОТЕЗЫ С ОДНИМ ЛИНЕЙНЫМ ОГРАНИЧЕНИЕМ

$$\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon};\ \mathbf{c}'\boldsymbol{\beta} = r$$

$$\hat{\boldsymbol{\beta}}_{OLS} \in \mathbf{N}\left(\boldsymbol{\beta}; \sigma^{\mathbf{2}}\left(\mathbf{X}'\mathbf{X}\right)^{-1}\right) \Rightarrow \mathbf{c}'\hat{\boldsymbol{\beta}}_{UR} \in \mathbf{N}\left(r; \sigma^{\mathbf{2}}\mathbf{c}'\left(\mathbf{X}'\mathbf{X}\right)^{-1}\mathbf{c}\right)$$

$$z = \frac{\mathbf{c}'\hat{\boldsymbol{\beta}} - r}{\sqrt{\sigma^{\mathbf{2}}\mathbf{c}'\left(\mathbf{X}'\mathbf{X}\right)^{-1}\mathbf{c}}} \in \mathbf{N}\left(\mathbf{0}; \mathbf{I}\right); \quad K = \frac{s^2\left(n-k\right)}{\sigma^2} = \frac{\mathbf{e}'\mathbf{e}}{\sigma^2} \sim \chi^2_{n-k}$$

$$t = \frac{z}{\sqrt{K/\left(n-k\right)}} = \frac{\mathbf{c}'\hat{\boldsymbol{\beta}} - r}{\sqrt{\frac{\mathbf{e}'\mathbf{e}}{n-k}\mathbf{c}'\left(\mathbf{X}'\mathbf{X}\right)^{-1}\mathbf{c}}} = \frac{\mathbf{c}'\hat{\boldsymbol{\beta}} - r}{\sqrt{s^{\mathbf{2}}\mathbf{c}'\left(\mathbf{X}'\mathbf{X}\right)^{-1}\mathbf{c}}} \sim t_{n-k}$$

## ПРОВЕРКА ГИПОТЕЗЫ С ОДНИМ ЛИНЕЙНЫМ ОГРАНИЧЕНИЕМ

$$\mathbf{y} = \mathbf{X\beta} + \boldsymbol{\varepsilon}; \ \mathbf{c}'\boldsymbol{\beta} = r$$

$$t = \frac{z}{\sqrt{K/(n-k)}} = \frac{\mathbf{c}'\hat{\boldsymbol{\beta}} - r}{\sqrt{\frac{\mathbf{e}'\mathbf{e}}{n-k}\mathbf{c}'(\mathbf{X}'\mathbf{X})^{-1}\mathbf{c}}} = \frac{\mathbf{c}\hat{\boldsymbol{\beta}} - r}{\sqrt{s^2\mathbf{c}(\mathbf{X}'\mathbf{X})^{-1}\mathbf{c}'}} \sim t_{n-k}$$

$$t^2 = \frac{\left(\mathbf{c}'\hat{\boldsymbol{\beta}} - r\right)^2}{s^2\mathbf{c}'(\mathbf{X}'\mathbf{X})^{-1}\mathbf{c}} = \frac{\left(\mathbf{c}\hat{\boldsymbol{\beta}} - r\right)'\left[\mathbf{c}'(\mathbf{X}'\mathbf{X})^{-1}\mathbf{c}\right]^{-1}\left(\mathbf{c}'\hat{\boldsymbol{\beta}} - r\right)}{\mathbf{e}'\mathbf{e}/(n-k)} \sim F(1; n-k)$$

$$Ex: \quad \mathbf{c}' = (0,\ldots 0,1_i,0\ldots 0) \Rightarrow F = \frac{\left(\widehat{\beta}_i - r\right)^2}{s^2\left[(\mathbf{X}'\mathbf{X})^{-1}\right]_{ii}} = \left[\frac{\widehat{\beta}_i - r}{s.e.\widehat{\beta}_i}\right]^2 = t_i^2$$

# ПРОВЕРКА ГИПОТЕЗЫ С ОДНИМ ЛИНЕЙНЫМ ОГРАНИЧЕНИЕМ

```
. reg S ASVABC SM SF

  Source |       SS       df       MS              Number of obs =     570
---------+------------------------------           F(  3,   566) =  110.83
   Model |  1278.24153     3  426.080508           Prob > F      =  0.0000
Residual |  2176.00584   566  3.84453329           R-squared     =  0.3700
---------+------------------------------           Adj R-squared =  0.3667
   Total |  3454.24737   569  6.07073351           Root MSE      =  1.9607

------------------------------------------------------------------------------
       S |      Coef.   Std. Err.       t     P>|t|       [95% Conf. Interval]
---------+--------------------------------------------------------------------
  ASVABC |   .1295006   .0099544     13.009   0.000       .1099486    .1490527
      SM |    .069403   .0422974      1.641   0.101       -.013676     .152482
      SF |   .1102684   .0311948      3.535   0.000       .0489967    .1715401
   _cons |   4.914654   .5063527      9.706   0.000       3.920094    5.909214
------------------------------------------------------------------------------
```

**Now we have added the highest grade completed by each parent. Does parental education have a significant impact? Well, we can see that a *t* test would show that *SF* has a highly significant coefficient, but we will perform the *F* test anyway. We make a note of *RSS*.**

## $F$ TESTS OF GOODNESS OF FIT

$$Y = \beta_1 + \beta_2 X_2 + \beta_3 X_3 + u \qquad RSS_1$$

$$Y = \beta_1 + \beta_2 X_2 + \beta_3 X_3 + \beta_4 X_4 + u \qquad RSS_2$$

$$H_0 : \beta_4 = 0$$

$$H_1 : \beta_4 \neq 0$$

$$F(\text{cost, d.f. remaining}) = \frac{\text{improvement} \;/\; \text{cost}}{\text{remaining unexplained} \;\Big/\; \text{degrees of freedom remaining}}$$

**The *F* test has the usual structure. We will illustrate it with an educational attainment model where *S* depends on *ASVABC* and *SM* in the original model and on *SF* as well in the revised model.**

# *F* TESTS OF GOODNESS OF FIT

```
. reg S ASVABC SM

  Source |       SS       df       MS                  Number of obs =     570
---------+------------------------------               F(  2,   567) =  156.81
   Model |  1230.2039      2  615.101949               Prob > F      =  0.0000
Residual | 2224.04347    567  3.92247526               R-squared     =  0.3561
---------+------------------------------               Adj R-squared =  0.3539
   Total | 3454.24737    569  6.07073351               Root MSE      =  1.9805

------------------------------------------------------------------------------
       S |      Coef.   Std. Err.       t     P>|t|       [95% Conf. Interval]
---------+--------------------------------------------------------------------
  ASVABC |   .1381062   .0097494     14.166   0.000       .1189567    .1572556
      SM |    .154783   .0350728      4.413   0.000       .0858946    .2236715
   _cons |   4.791277   .5102431      9.390   0.000        3.78908    5.793475
------------------------------------------------------------------------------
```

**Here is the regression of *S* on *ASVABC* and *SM*. We make a note of the residual sum of squares.**

# $F$ TESTS OF GOODNESS OF FIT

```
. reg S ASVABC SM SF

  Source |       SS       df       MS              Number of obs =     570
---------+------------------------------           F(  3,   566) =  110.83
   Model |  1278.24153     3  426.080508           Prob > F      =  0.0000
Residual |  2176.00584   566  3.84453329           R-squared     =  0.3700
---------+------------------------------           Adj R-squared =  0.3667
   Total |  3454.24737   569  6.07073351           Root MSE      =  1.9607

------------------------------------------------------------------------------
       S |      Coef.   Std. Err.       t     P>|t|       [95% Conf. Interval]
---------+--------------------------------------------------------------------
  ASVABC |   .1295006   .0099544     13.009   0.000       .1099486    .1490527
      SM |    .069403   .0422974      1.641   0.101       -.013676     .152482
      SF |   .1102684   .0311948      3.535   0.000       .0489967    .1715401
   _cons |   4.914654   .5063527      9.706   0.000       3.920094    5.909214
------------------------------------------------------------------------------
```

$$F(1,566) = \frac{(2224.0 - 2176.0)/1}{2176.0/(566)} = 12.49 \qquad F_{\text{crit},0.1\%} = 11.38$$

$$3.535^2 = 12.496 \qquad t_{\text{crit},0.1\%} = 3.373 \qquad 3.373^2 = 11.377$$

**If all the variables are correlated, it is possible for all of them to have low marginal explanatory power and for none of the *t* tests to be significant, even though the *F* test for their joint explanatory power is highly significant.**

**If this is the case, the model is said to be suffering from the problem of multicollinearity**

# пример теста на линейные ограничения

- reg lnQ lnL lnK

```
      Source |       SS       df       MS              Number of obs =      15
-------------+------------------------------           F(  2,    12) =  186.81
       Model |  .061682789     2  .030841395           Prob > F      =  0.0000
    Residual |  .001981101    12  .000165092           R-squared     =  0.9689
-------------+------------------------------           Adj R-squared =  0.9637
       Total |   .06366389    14  .004547421           Root MSE      =  .01285

------------------------------------------------------------------------------
         lnQ |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
         lnL |   .7575309   .7073246     1.07   0.305    -.7835969    2.298659
         lnK |    .188014    .138675     1.36   0.200    -.1141328    .4901608
       _cons |   .5006224   4.480004     0.11   0.913    -9.260468    10.26171
------------------------------------------------------------------------------
. test _b[lnK]+_b[lnL]=1

 ( 1) _b[lnK]+_b[lnL]=1

       F(  1,    12) =    0.01
            Prob > F =    0.9255
```

Конец лекции